ON A CHARACTERISTIC OF GAS RADIATION IN THE COMPLETE SPECTRUM

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The quantity ε_0, which has the meaning of the fraction of black radiation intensity corresponding to the spectrum of surface radiation of a gas body, is investigated.

The gas volume in chambers is assumed isothermal with effective temperature T in approximate computations of heat exchange by radiation. In the simplest case of a gas in a grey shell, the heat-exchange computation is carried out by exact formulas [1]. These formulas have been simplified somewhat in [2] because of regrouping of terms. In the cases of a multizone grey shell, a nongrey shell, gas turbidity, and others, the computation should be carried out over the spectrum bands using the band models. But because of the rough approximation due to the introduction of the effective temperature, this complex path may turn out to be illogical. A simple method consisting of combining the bands and extracting the "windows" of the spectrum is recommended. The need for the radiation characteristic ε_0 originates here. The quantity ε_0 has the meaning of the fraction of black radiation power corresponding to the spectrum of surface radiation of a gas body. The difference $1-\varepsilon_0$ expresses the radiation intensity in the windows of the spectrum. The transmissivity of a gas for a beam with a black spectrum at the same temperature is

$$D = 1 - \varepsilon(x, T) = \varepsilon_0 D_0 + 1 - \varepsilon_0. \quad (1)$$

The introduction of the quantity ε_0 permits taking account of radiation in the bands and windows of the spectrum separately by means of the optical constants averaged in each part of the spectrum. Quite different quantities are used in the literature as ε_0. The best known is the quantity ε_∞ called the degree of blackness of an infinitely thick gas, introduced by Shak. Let us show that the values of ε_∞ in the literature are far from the real values, which are quite close to one, and then in any case the quantity ε_∞ does not correspond to the designation ε_0. We provide all the illustrations for the case of carbon dioxide gas.

Let us consider the formula for the degree of blackness of CO_2 derived by Shak by means of three spectrum band parameters [3]. Up to now this formula should be considered most exact for $0 < x \le 0.4$ mat [4]. From the formula for $x \to \infty$ there is obtained

$$\varepsilon_{\infty,1} = 1.1630\,(K_{01} + K_{02} + K_3)/\sigma T^4,$$

where

$$\sigma = 5.77\cdot10^{-8}\ \text{W/m}^2\cdot\text{deg}^4; \quad K_{0j} = q_{0\lambda_j}\,\Delta\lambda_{ej};$$

$$q_{0\lambda_j} = 3.17\cdot10^8\,\frac{\lambda_j^{-5}}{\exp\left(\dfrac{1.43}{\lambda_j T}\cdot10^4\right)-1};$$

$$\Delta\lambda_{e1} = \Delta\lambda_1\left(1+0.026\,\frac{t}{100}\right);\quad \Delta\lambda_{e2} = \Delta\lambda_2\left(1+0.031\,\frac{t}{100}\right);$$

$$\Delta\lambda_{e3} \equiv \Delta\lambda_3.$$

Values of λ_j and $\Delta\lambda_j$ are given in Table 1. Shak here incorporated the "zero" 2.05 μ band in the band 1. The adjacent weak bands were also adjoined to the bands 2 and 3. As the gas thickness increases, the weak

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TABLE 1. Parameters of the Spectrum Bands for Computation of the Quantities $\varepsilon_{\infty,1}$ by the Shak Formula

i	λ_j, μ	$\Delta\lambda_j$, μ
1	2,7	0,30
2	4,3	0,412
3	15,0	4,6

TABLE 2. Parameters of the Spectrum Bands for Computation of the Quantity $\varepsilon_{\infty,2}$ by Four Bands

i	λ_j, μ	$\Delta\lambda_j$, μ
0	2,05	0,21
1	2,7	0,27
2	4,3	0,412
3	16,0	4,6

bands become strong and should be taken into account separately. As the 2.05 μ band is extracted, the computation of $\varepsilon_{\infty,2}$ is carried out by a four-term formula with the parameters indicated in Table 2. Extraction of the band as x increases can be continued. However, this means is meaningless since the band contours in the spectrum of gas surface radiation gradually diffuse, the bands overlap, and the windows vanish. The quantities $\varepsilon_{\infty,3}$ obtained by extrapolating the function $\varepsilon(x)$ on a VTI nomogram are used in the Soviet literature. The quantities $\varepsilon_{\infty,i}$ obtained by diverse methods are compiled in the sketch. Let us add that one value $\varepsilon_\infty = 0.23$ was taken in [2] for the $0 < t < 1600°C$ temperature band. Despite the discrepancy, these data could be used as empirical since the actual value of ε_∞ which is close to one, is useless. However, there is another defect. The quantities $\varepsilon_{\infty,i}$ depend only on the temperature while the characteristic ε_0 should have a strong dependence on the beam length. For example, for $t = 800°C$ it varies between 0.039 and 1. Indeed, the degree of blackness in thin layers ($x = 0.03$ mat) is determined by the one band 4.3 μ in practice. The optical constants should be extracted at 4.3 μ in computing the reflection and scattering in the gas spectrum. But as the gas thickness increases the weak bands are strengthened, their contribution to the radiation on the gas surface increases, and the quantity ε_0 also increases correspondingly, but the optical constants should be averaged over all the strong bands.

The quantity ε_∞ is avoided in [5], but ε_X and D_X, with the literal definitions given above for ε_0 and D_0, are introduced, respectively. The formulas

$$\varepsilon_x = \varepsilon^2(x)/[2\varepsilon(x) - \varepsilon(2x)];\; D_x = 1 - \varepsilon(x)/\varepsilon_x$$

are recommended; or written differently

$$D_x = \frac{\varepsilon(2x) - \varepsilon(x)}{\varepsilon(x)}, \tag{2}$$

$$\varepsilon_x = \varepsilon(x)/(1 - D_x). \tag{3}$$

Formula (2) yields a simple meaning for the quantity D_X. Let us compare two adjacent sections with identical thickness x. A beam having its source in section 1 passes through section 2. The quantity D_X is the transmissivity of section 2 for the beam with spectrum on the boundary of section 1. From (3) we obtain $\varepsilon_X = a_{02}/a_{12}$, where a_{02} is the absorptivity of section 2 with respect to the black beam, and a_{12} is the same for the radiation of section 1. The selection of (2) has not sufficient foundation. It is logical that D_X describes the transmissivity of a section of length x. The selection of a section of gas as the beam source is logical, but the thickness of the section 1 governing the beam spectrum has no rigorous foundation. In determining ε_X and D_X Hottel admits even other pairs of values, $\varepsilon(x)$ and $\varepsilon(3x)$, say, which determine ε_X by more complex formulas. In place of ε_X in [6, 7], the quantity ε_* is used with the distinction that the section 1 is assumed elementary. Then

$$D_* = \alpha_c^{-1}\,\partial\varepsilon/\partial x;\; \alpha_c = (\partial\varepsilon/\partial x)_{x=0};\; \varepsilon_* = \varepsilon/(1 - D_*). \tag{4}$$

Formulas (1) and (3) remain unchanged. Using the quantity ε_* just as the quantity ε_X has no strict foundation, but both quantities correspond to the properties of the characteristic ε_0 while the quantity ε_∞ does not sustain criticism.

Let us turn to an interpretation of the quantity ε_0 on the basis of a spectrum model. Using discrete values of the Planck function (J_{0j}) for the centers of the bands, the degree of blackness is

$$\varepsilon(x, T) = \frac{\pi}{\sigma T^4}\sum_j J_{0j}\int_{\Delta\omega}\varepsilon_\omega d\omega;\; \varepsilon_\omega = 1 - \exp(-\alpha_\omega x). \tag{5}$$

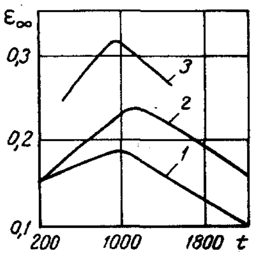

Fig. 1. Degree of blackness of an infinitely thick gass according to different sources (t, °C): 1) $\varepsilon_{\infty,1}$, according to Shak, taking a account of three bands including adjacent weak bands; 2) $\varepsilon_{\infty,2}$, according to Shak, taking account of four spectrum bands; 3) $\varepsilon_{\infty,3}$, values used in the Soviet literature.

Following Shak, let us rearrange the elementary intervals of the wave numbers so that the absorption coefficient will vary monotonically. Let us assume that the contour obtained is described by an exponential law

$$\alpha_\omega = \alpha_0 \exp(-B\nu);\ \ \nu = |\omega - \omega_0|.$$

The integral in (5) is reduced to

$$\int_{\Delta\omega_j} \varepsilon_\omega d\omega = \bar{A}_j \Delta\bar{\omega}_j;\ \bar{A}_j = 0.577216 + \ln \alpha_0 x + E_1(\alpha_0 x).$$

More exact spectrum models yield other formulas for the quantity $\bar{A}_j$. However, it is impossible to obtain formulas for the characteristic ε_0 of interest to us therefrom since these models refer to the function $\alpha_\omega(\omega)$ describing volume radiation directly, while the quantity ε_0 describes the surface radiation of the gas body. The spectrum on the surface bounding the volume should be examined. In contrast to $\Delta\bar{\omega}_j$, its bandwidth will depend on the optical thickness of the gas as well as on other parameters $\Delta\omega_{ej}(x, T, P, p)$. Let us assume that the function $\varepsilon_\omega(\omega)$ varies according to the symmetric exponential law

$$\varepsilon_\omega = \varepsilon_{w_0} \exp(-b\nu).$$

Then the integral in (5) becomes

$$\int_{\Delta\omega_j} \varepsilon_\omega \, d\omega = 2 \int_0^\infty \varepsilon_{\omega_0} \exp(-b\nu)\, d\nu = \varepsilon_{\omega_0} \Delta\omega_{ej}.$$

The formula for the characteristic ε_0 is obtained under the condition of total absorption at the centers of the bands, i.e., $\varepsilon_{\omega_0} = 1$:

$$\varepsilon_0(x, T) = \frac{\pi}{\sigma T^4} \sum_j J_{0j} \Delta\omega_{ej}. \tag{6}$$

The other band contours reduce to the same formula but with other values for the effective bandwidth. The actual contour is unknown and it is quite difficult to find it, even more so since it depends on parameters.

Formula (6) yields a simple interpretation of the characteristic ε_0 because of the clarity of the concept of effective bandwidth. However, it is quite difficult to find the function $\Delta\omega_{ej}(x, T, P, p)$, hence (6) does not solve the problem. The difficulties described in determining the quantity ε_0 are explained by the nature of the spectrum itself. Strictly speaking, there are no "windows" in a spectrum; their extraction to simplify the computation is admissible as an approximation. The characteristic ε_0 is, correspondingly, also approximate. The extreme values of ε_0, the minimum and maximum, are known. The intermediate values of the function $\varepsilon_0(x)$ at a given temperature should be refined by comparing the exact and approximate solutions of a simple problem.

In conclusion, let us note that all the relationships have been written down for equal temperatures of a black emitter (T_0) and of the gas. If the temperatures are not equal, or the spectrum of the beam entering the gas is not black, then the absorptivity $a(x, T, T_0)$ should be used throughout in place of the degree of blackness.

NOTATION

$x = \int p dl$	is the beam length referred to the partial pressure p;
P	is the total pressure;
$T = t + 273$, °K	is the effective gas temperature;
T_0	is the temperature of the black source;
λ	is the wavelength;
$\varepsilon(x, T)$	is the degree of blackness;

ε_0 is the desired characteristic;
D is the gas transmissivity for a black beam;
D_0, D_x, D_* are the same for beams of the parts ε_0, ε_x, ε_* of the black body spectrum;
$\varepsilon_{\infty,i}$ is the degree of blackness of an infinitely thick gas for various sources;
α_ω, $(\text{mat})^{-1}$ is the spectral absorption coefficient;
ε_ω is the spectral degree of blackness;
J_{0j} is the Planck's function for the center of band j on the wavenumber scale;
ω, cm^{-1} is the wave number;
$\Delta\omega$ is the bandwidth in the volume radiation spectrum;
$\Delta\omega_{ej}$ is the effective bandwidth in the surface radiation spectrum;
$\Delta\lambda_{ej}$ is the same on the wavelength scale;
b, B are the coefficients with a length dimensionality;
$\bar{A}$ is the mean absorption in the interval $\Delta\bar{\omega}_j$;
$\Delta\bar{\omega}_j = 2/B$;
$\nu = |\omega - \omega_0|$;
ω_0 is the center of the band;

$$E_1(u) = \int_1^\infty \tau^{-1} \exp(-u\tau) d\tau.$$

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